Image Reconstruction using RAPID Algorithm with Time-Frequency Analysis

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***Abstract*—The detection of cracks is crucial in some industries, such as petroleum and aerospace. In order to detect these cracks, a modification to the Reconstruction Algorithm for Probabilistic Inspection Damage (RAPID) has been proposed. This modification consists of comparing the time-frequency content of the signals instead of only analysing their waveforms. This analysis has some advantages over the usual analysis, such as filtering the signal and observing the behaviour of defects with different frequencies. The results of this experiment show the ability of this new approach to localize defects agreeing with the real position of the defect. However, the shape detection of this approach needs to be improved. Nevertheless, the modification presented in this paper shows great potential and with further work it is possible to obtain better results.**

***Index Terms*—Frequency-domain analysis, Nondestructive testing, Time-domain analysis, Ultrasonic imaging**

I. Introduction

The field of NDT is an interdisciplinary field focused on evaluating the status and properties of materials, components or systems without affecting their future use [1]. Among these techniques exists one called ultrasonic testing, which uses ultrasonic waves that travel through the material to detect flaws or defects. These ultrasonic waves can be Bulk waves, Lamb waves, Rayleigh waves, etc. Among waves, Lamb waves are widely used for their sensibility and different modes [2]. After their propagation, these waves are acquired and processed using tomographic algorithms which create a pseudo image that is used to detect the position and shape of the defect or flaw in the material.

Ultrasonic Wave tomography has been researched widely in recent years. Hildebrand [3] and Leonard *et al* [4] investigated lamb wave tomography for pipes. Giurgiutiu *et al* [5] studied the ultrasonic waves in beams and plates using an embedded piezoelectric wafer active sensor (PWAS). Rathod and Mahapatra [6] monitored the growth of corrosion using wavelets and calculated a damage index for the different frequencies. Zhenkun [7] estimated the Time of flight of a Multi-superimposed Ultrasonic signal using wavelet denoising and the Hilbert Transform to improve accuracy and reduce computational time. Rajagopalan *et al* [8] designed a new configuration of PZT called single transmitter multi-receiver (STMR) which uses a phased addition algorithm in the wavenumber domain, and their reconstructions were of good quality. Xiang *et al* [9] compared different tomographic imaging techniques, such as Filtered Backprojection (FBP), Algebraic Reconstruction Technique (ART) and Reconstruction Algorithm for Probabilistic Inspection of Damage (RAPID), and their advantages and disadvantages in reconstruction fidelity, quality and efficiency. Moreover, Xiang presents a discussion about different arrays of sensors and their resolution. Among the algorithms presented by Xiang, the RAPID algorithm has the advantages of simplicity and speed, whereby several studies have been carried out with it [10]–[12]. Sheen [13] presented a study to calculate the parameter β used in the RAPID algorithm to obtain better results. Dziendzikowski [14] used the RAPID algorithm to localize impact damage on composite structures and compared its results with other NDT techniques. S. Wang [15] compared the image qualities of the three commonly used array of sensors (circular, rectangular and parallel) using the RAPID algorithm to show the different characteristics of the arrays. The RAPID algorithm performs an analysis that creates a map of probabilities. These probabilities are consequences of the difference between two signals, the reference signal and the other signal acquired after a certain time. Any difference between the two signals will be attributed to a defect. However, the RAPID algorithm only provides a pseudo image of the tested area. The aim of this paper is to propose a modification to the RAPID algorithm. The modification consists of applying the Gabor transform to the signals in order to apply the RAPID algorithm to time-frequency data instead of the usual time data.

II. RAPID Algorithm

The RAPID algorithm detects variations or features induced in the ultrasonic waves by structural flaws. The features are extracted by measuring the differences between the normal or reference signal and the damaged signal. The signals acquired just after the sensors are installed are used as the reference signals. Any other signals collected afterwards will be used as the damaged signal. The damaged signal is compared with the reference signals to identify possible variations. The variations are attributed to a defect because the signals are measured in the same environment. The severity of the variations in the signals is considered as the probability of the presence of a defect. This algorithm has an advantage in that it can be used for non-uniform surfaces or with multiple elements, for example, a wing panel as observed in [11]. Furthermore, this algorithm can be used to detect the growth of the defect. The RAPID algorithm measures the variations in the signals by a signal difference coefficient (SDC) [10]. The SDC is calculated using a correlation coefficient ρ given by:

$$\rho = \frac{C_{XY}}{\sigma_X \sigma_Y} \tag{1}$$

where X is the reference signal, Y is the damaged signal, C_{XY} is the covariance of X and Y, and σ_X and σ_Y are the standard deviation of X and Y, respectively. Finally, the SDC is calculated by:

$$SDC = 1 - \rho \tag{2}$$

The proposed method performs the SDC to the time-frequency data of the signal instead of performing it only to its waveform. This modification measures the changes between the reference spectrum and the current spectrum over time. Thus, the modified algorithm can be used to visualise the behaviour of different frequencies through time. The time-frequency data is calculated using the Gabor transform [16] given by:

$$G_x(\tau,\omega) = \int_{-\infty}^{\infty} x(t) e^{-\pi(t-\tau)^2} e^{-j\omega t} dt \tag{3}$$

where τ is the width of the gaussian function, ω is the angular frequency and $x(t)$ is the time data. The data obtained with the Gabor transform will be analysed by the SDC resulting in the variations generated by the defect.

Finally, the result of the SDC will be introduced into the main equation of the RAPID algorithm [11], which is:

$$P(x,y) = \sum_{i=1}^{N-1} \sum_{j=i+1}^{N} A_{ij} \left[\frac{\beta - R_{ij}(x,y)}{\beta - 1} \right] \tag{4}$$

where $P(x,y)$ is the reconstructed region, (x,y) is the defect probability at coordinates "x,y", A_{ij} is the SDC of the sensor pair S_{ij} (transmitter i, receiver j), β is the scaling parameter that controls the effective elliptical distribution area and $R_{ij}(x,y)$ is the elliptical distribution between the receiver and transmitter. This elliptical distribution is used to emphasise the effect of the wave to the points in the direct path between the sensors. The elliptical distribution is observed in Fig. 1.

Also, the elliptical distribution is calculated by:

$$R_{ij}(x,y) = \begin{Bmatrix} RD_{ij}(x,y), & when & RD_{ij}(x,y) < \beta \\ \beta, & when & RD_{ij}(x,y) \geq \beta \end{Bmatrix} \tag{5}$$

where

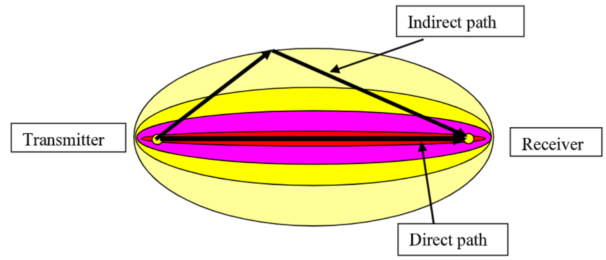

Fig. 1. Elliptical distribution

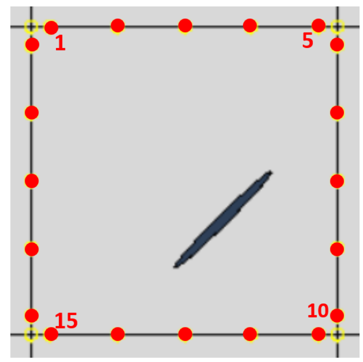

Fig. 2. Tested area with defect

$$RD_{ij}(x,y) = \frac{\sqrt{(x-x_i)^2 + (y-y_i)^2} + \sqrt{(x-x_j)^2 + (y-y_j)^2}}{\sqrt{(x_j-x_i)^2 + (y_j-y_i)^2}} \tag{6}$$

where (x,y) are the coordinates of the evaluated point, (x_i, y_i) are the coordinates of the transmitter and (x_j, y_j) are the coordinates of the receiver.

The final result of the RAPID algorithm is a map of probabilities, which indicates the possible location of a defect.

III. Experimentation

The proposed experiment consists of a simulation of two sheets (damaged and undamaged) of aluminium of $0.3m \times 0.3m \times 0.001m$. This size will prevent the waves from bouncing at the edges. The sheets have a square array of 20 sensors surrounding the tested area. The tested area is a square of $0.08m$ per side in the center of the sheet. The defect is a elliptical crack near the center of the sheet. The sheet of aluminium with the position of the sensors can be observed in Fig. 2.

The applied signal is a windowed sinusoidal signal of five cycles at 250 kHz, as observed in Fig. 3. This frequency was selected because this frequency only generates A0 and S0 wave modes, producing cleaner signals that are easier to process. This can be observed in Fig. 4 and 5.

The experiment was simulated in Abaqus/CAE. The wave is generated using a Matlab code which generates a table

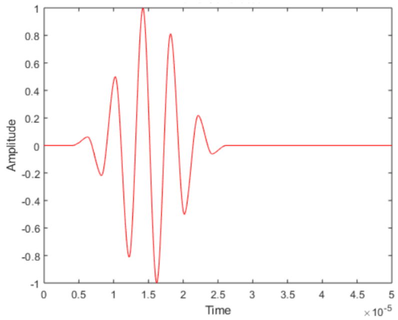

Fig. 3. Windowed sinusoidal signal of 5 cycles at 250 kHz

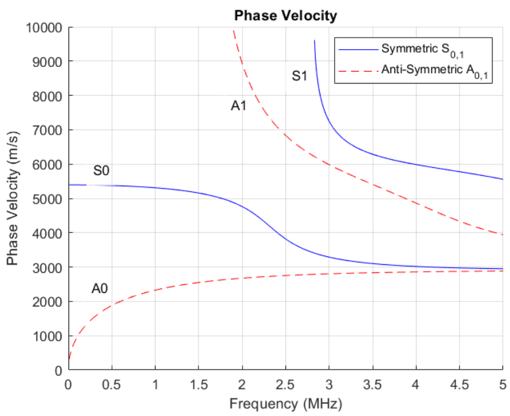

Fig. 4. Phase Velocity of Aluminium

with the values of amplitude. These values are introduced in Abaqus/CAE to generate the signal.

The experiment consists of a sweep, where for each iteration the signal will be applied in a different sensor while the rest of the sensors will read the propagated wave in the material. The sweep will provide information about the tested area. The data collected by the sensors will be used in Matlab, where the modified RAPID algorithm has been developed.

The first step in Matlab is to generate the Gabor transform of the different signals. Afterwards, instead of using the full spectrum, only the frequencies around 250 kHz are selected. The resulting data is shown in Fig. 6.

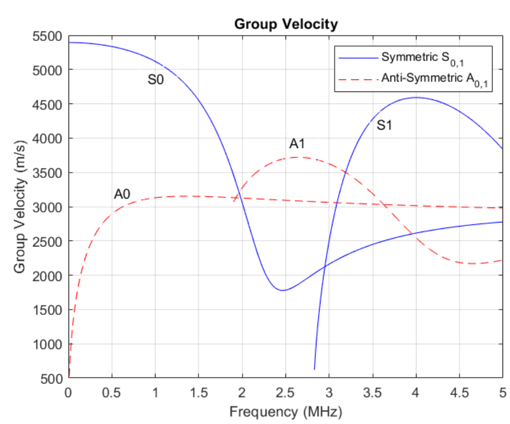

Fig. 5. Group Velocity of Aluminium

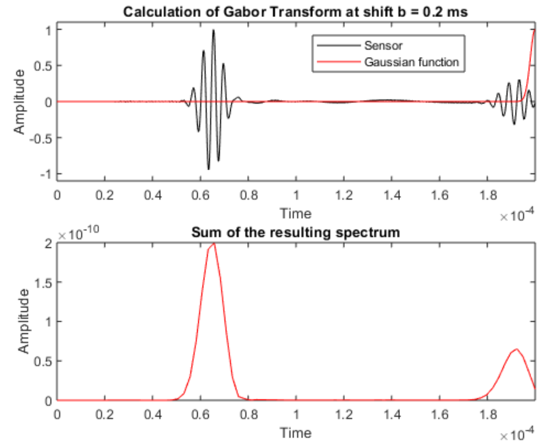

Fig. 6. Data obtained after applying Gabor Transform

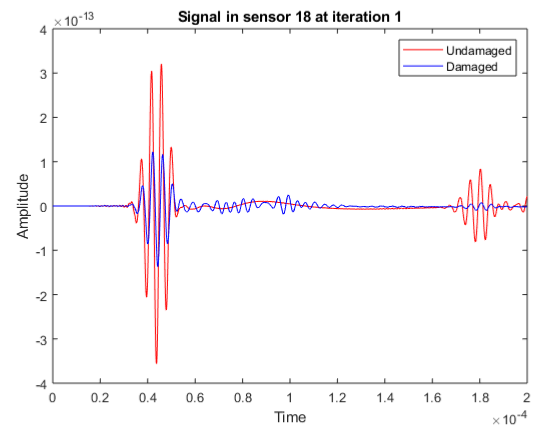

Fig. 7. Comparison between reference signal and damaged signal in Sensor 18

With this transformation, the algorithm only analyses the frequencies around the main frequency, eliminating the possible noise that can be introduced by the environment. After this transformation, the SDC is used to detect the variations in the two signals. One example of this variation can be observed in Fig. 7. Finally, the main equation of the RAPID algorithm is applied to generate the map of probabilities.

IV. Results

The variations introduced by the defect can be observed in Fig. 8. In this example, the defect has reduced the amplitude of the wave and has introduced a small oscillation. In this example, the attenuation and the oscillation will be attributed to a possible defect between the ray path of sensors 18 and 1. All the different variations introduced by the defect in the different waves will be processed by the RAPID algorithm. These variations are used to detect the shape and locality of the defect, resulting in the map of probabilities shown in Fig.

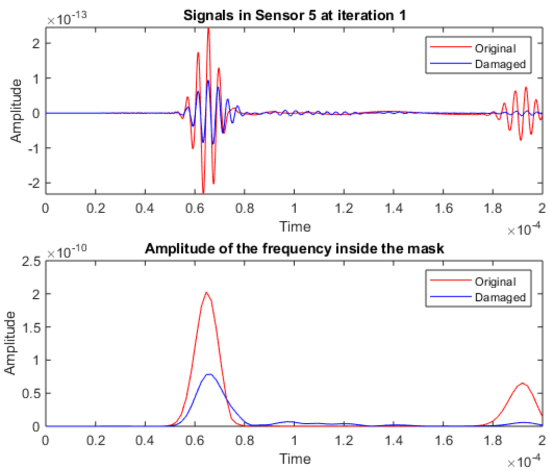

Fig. 8. Comparison between reference signal and damaged signal in the time-frequency domain in Sensor 5

9. This map gives a good approximation of the position of the simulated crack, which leads to good defect detection. The good approximation is a result of selecting certain frequencies during the process of time-frequency analysis, that will lead to a simpler signal in comparison to the original and additionally it will be a filtered signal. However, in this experiment, only the frequencies around 250 kHz were selected because no harmonics were found in the spectrums. Also, the resulting map needs to improve shape detection in order to achieve better results.

V. Conclusions and Further work

A modification to the RAPID algorithm has been presented. This modification consists of analysing the time-frequency content of the signals, instead of only analysing their waveform. The result of this modification is satisfactory but it can also be improved. One of the reasons for the good result is that the signals have been filtered and the noise has been removed. However, the next step of this algorithm could be to analyse the behaviour of the harmonics, which could provide extra information about the defect. Thereby, new simulations with different defects are needed to obtain the new data. Further work includes testing the algorithm using real experimentation. Currently, an experimental model with a pipe is being constructed to validate the algorithm. This set up consists of a pipe with two rings of sensors at both ends, and a crack between the rings. This model will explore different cracks to visualise the behaviour of the different frequencies in the pipe and identify different ways of improving this algorithm.

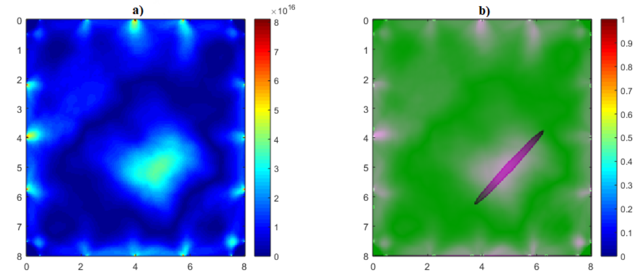

Fig. 9. **a)** Map of probabilities **b)** Map of probabilities with the real position of the crack

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